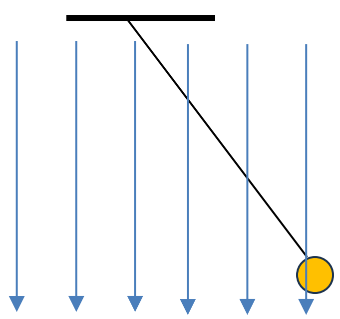
Teacher notes
Topic D

Electric effects and SHM

1. Consider the pendulum in the figure below. The bob has mass m and a positive charge q and is placed in a region of a vertical uniform electric field E. The length of the string is L.

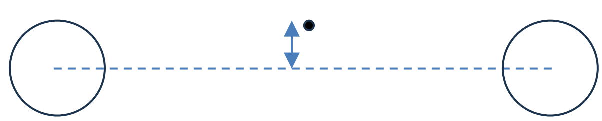


What is the period of small oscillations of the pendulum?

The electric force on the bob is a vertical, downward force qE and is added to the weight. So effectively it is as is we have a bob with a weight of $qE+mg$ which again is effectively the same as the motion of a mass m in a gravitational field of gravitational field strength $g+\frac{qE}{m}$.

Hence the period is $T=2\pi\sqrt{\frac{L}{g+\frac{qE}{m}}}$.

2. Two equal, positive charges Q are a distance $2a$ apart (center-to-center). A negative point charge of magnitude q is a placed a distance x above the midpoint of the line joining the centers. The distance x is much smaller than the distance a.



Explain why the point charge will perform simple harmonic oscillations and determine the period.

$$F_{\text{net}} = \frac{kQq}{a^2+x^2} \times 2 \times \frac{x}{\sqrt{a^2+x^2}} = \frac{2kQq}{(a^2+x^2)^{\frac{3}{2}}}x$$

Since $x << a$, we may ignore the *x* in the denominator and so $F_{\text{net}} \approx \frac{2kQq}{a^3}x$. The force is directed towards the mid-point of the line joining the centers. At the midpoint the force is zero so this is the equilibrium position. The net force is directed towards the equilibrium position and is proportional to the displacement from equilibrium and so we will have simple harmonic oscillations. The acceleration obeys

$$a \approx -\frac{2kQq}{ma^3}x$$

Hence $\omega^2 = \frac{2kQq}{ma^3}$ and so $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{ma^3}{2kQq}}$.

Discuss what would happen if the point charge was also positive.

3. The setup in 2 is changed as follows: the spherical charges *Q* are replaced by equal masses *M* and the point charge *q* by a point mass *m*. Discuss the motion of the point mass.

The situation is identical to the previous problem:

$$F_{\text{net}} = \frac{GMm}{a^2+x^2} \times 2 \times \frac{x}{\sqrt{a^2+x^2}} = \frac{2GMm}{(a^2+x^2)^{\frac{3}{2}}}x \approx \frac{2GMm}{a^3}x$$

$$a = -\frac{2GM}{a^3}x$$

$$\omega^2 = \frac{2GM}{a^3}$$

$$T = 2\pi\sqrt{\frac{a^3}{2GM}}$$

Can you think of any relevance of the result above to the stability of the rings of planets such as Saturn?